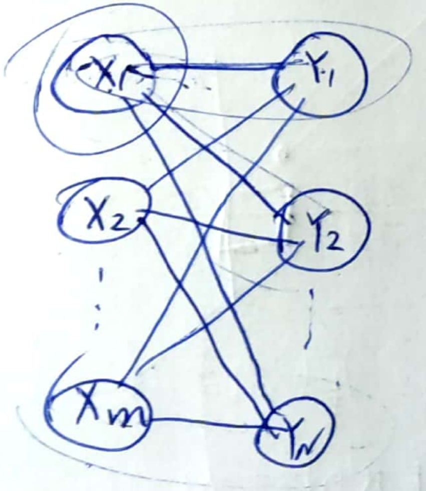


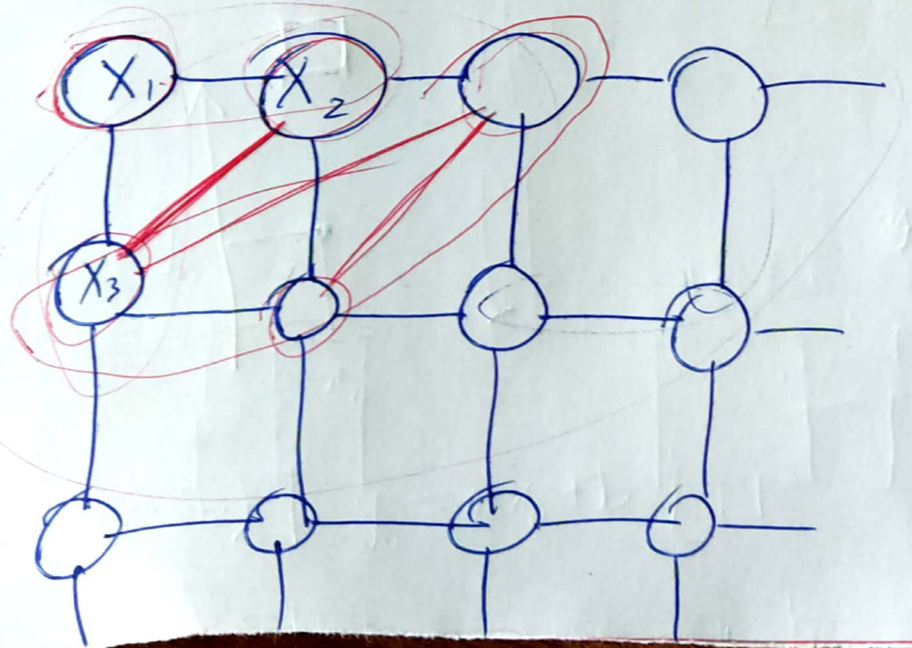
$$P(X_1 - X_m, Y_1 - Y_n) = \frac{1}{Z} \prod_{i=1}^{m} \prod_{j=1}^{n} \phi_{ij}(X_i, Y_j)$$

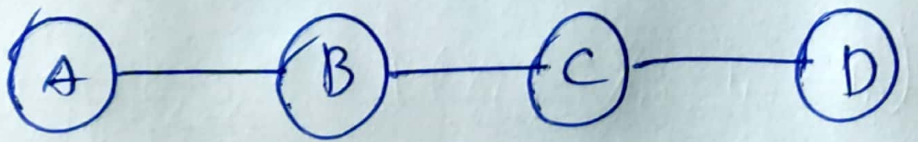
Eliminate X_1

ψ

$$\sum_{X_1} \underbrace{\phi_{11}(X_1, Y_1)\, \phi_{12}(X_1, Y_2) \cdots \phi_{1m}(X_1, Y_n)}$$

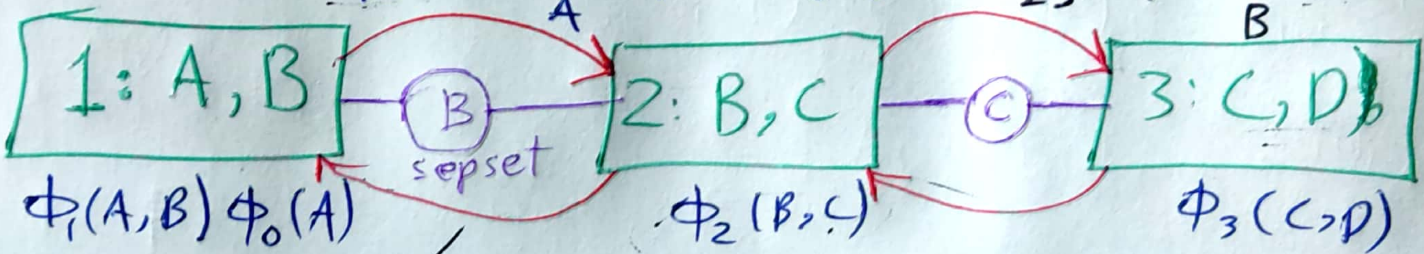
$$\sum_{X_1} \psi(X_1, Y_1, Y_2, \ldots, Y_n)$$





$$P(A,B,C,D) = \phi_1(A,B)\,\phi_2(B,C)\,\phi_3(C,D)\,\phi_0(A)$$

$$\tau_{12}(B) = \sum_A \phi_1(A,B)\,\phi_0(A) \qquad \tau_{23}(C) = \sum_B \tau_{12}(B)\,\phi_2(B,C)$$



$$\tau_{21}(B) = \sum_C \phi_2(B,C)\,\tau_{32}(C) \qquad \tau_{32}(C) = \sum_D \phi_3(C,D)$$

belief

$$\beta(B,C) = \phi_2(B,C)\,\tau_{12}(B)\,\tau_{32}(C)$$

$$= \phi_2(B,C)\left(\sum_A \phi_1(A,B)\,\phi_0(A)\right)\left(\sum_D \phi_3(C,D)\right)$$

$$= \sum_A \sum_D \phi_0(A)\,\phi_1(A,B)\,\phi_2(B,C)\,\phi_3(C,D)$$

$$= \sum_A \sum_D P(A,B,C,D) = P(B,C)$$

$$\beta(C,D) = \phi_3(C,D)\,\tau_{23}(C) =$$

$$= \phi_3(C,D) \boxed{\sum_B \tau_{12}(B)\,\phi_2(B,C)}$$

$$= \phi_3(C,D) \sum_B \boxed{\sum_A \phi_1(A,B)\,\phi_0(A)}\ \phi_2(B,C)$$

$$= \sum_A \sum_B \phi_3(C,D)\,\phi_1(A,B)\,\phi_0(A)\,\phi_2(B,C)$$

$$= \sum_A \sum_B P(A,B,C,D) = P(C,D)$$

$$\beta(B) = \tau_{12}(B)\,\tau_{21}(B) = \boxed{\sum_A \phi_1(A,B)\,\phi_0(A)}\ \boxed{\sum_C \phi_2(B,C)\,\tau_{32}(C)}$$

$$= \boxed{\sum_A \phi_1(A,B)\,\phi_0(A)}\ \boxed{\sum_C \phi_2(B,C)\ \boxed{\sum_D \phi_3(C,D)}}$$

$$= \sum_A \sum_C \sum_D \phi_1(A,B)\,\phi_0(A)\,\phi_2(B,C)\,\phi_3(C,D)$$

$$= \sum_A \sum_C \sum_D P(A,B,C,D) = P(B)$$

$$P(D \mid C=c) = \frac{P(D, C=c)}{P(C=c)} = \frac{P(D, C=c)}{\sum_{D'} P(D', C=c)} = \frac{\frac{1}{2}\tilde{P}(D, C=c)}{\sum_{D'} \frac{1}{2}\tilde{P}(D', C=c)}$$

$$= \frac{\tilde{P}(D, C=c)}{\sum_{D'} \tilde{P}(D', C=c)} = \frac{\beta(D, C=c)}{\sum_{D'} \beta(D', C=c)}$$

$$P(C, D \mid A=a) = \frac{P(C, D, A=a)}{\underbrace{\sum_{C'}\sum_{D'} P(C', D', A=a)}_{P(A=a)}} = \frac{\tilde{P}(C, D, A=a)}{\sum\sum P(C, D, a)}$$

$$\tilde{P}(C, D, A=a) = \sum_{B} \phi_1(a, B)\, \phi_2(B, C)\, \phi_3(C, D)\, \phi_0(a)$$

$$\sum_{B}\sum_{A} \underbrace{\phi(A, B)}\, \phi_2(B, C)\, \phi_3(C, D)\, \underbrace{\phi_0(A)\, 1(A=a)}$$